Modelling of crater formation

Key stage 5

The scenario

In 2018, scientists will be landing a seismometer on Mars and expect to start detecting signals from meteorite impacts early in 2019. Satellite images from Mars can be used to identify new impact craters; if we can locate these new craters are and work out how much energy it took to create them, scientists will be able use the seismic signals to discover more about the Martian interior.

For this exercise, you are working in one of the project teams for this mission. Each team has been tasked to investigate high velocity meteorite impacts on the surface of Mars.

Your group have decided to simulate impact craters using low-velocity experiments. Your group has been told that the surface of Mars is very powdery like flour.

A homework could be to research the mathematical modelling of crater formation.

Crater activity: mathematical modelling of impactors

The impactors

For any of the investigations you will need to collect together spherical objects of different diameters. The smallest ball doesn't want to have a diameter much less than 1 cm (a marble); anything much smaller and the object tends to burrow rather than produce a crater.

The best impactor has been found to be a wooden ball (the density is a key factor) of radius around 1.75 cm and mass 30 g.

The landing area

Prepare the impactor landing area. This could be a deep baking try or a cardboard box — something with high sides works well to prevent ejecta escaping. The landing area should be at least 30 cm by 30 cm. The landing material is a key factor to consider; flour or fine sand are both possibilities.

Flour has been found to be the best material. Very small glass beads model real impact surfaces well. The surface can also be given a spray from an indoor plant sprayer/mister; the surface forms a realistic crust.

Learning objectives

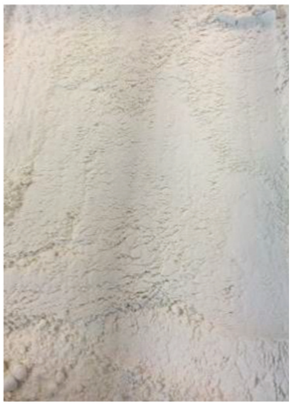
- understand the role impactors have in crater formation
- understand the importance of controlling variables in producing reliable data
- describe the energy transfers as an object falls and impacts
- use the correct equations to calculate impact velocities
- use data to accurately plot line graphs
- analyse data and graphs to report on patterns and relationships
- building a mathematical model
- evaluate data and experimental procedure and comment on reliability

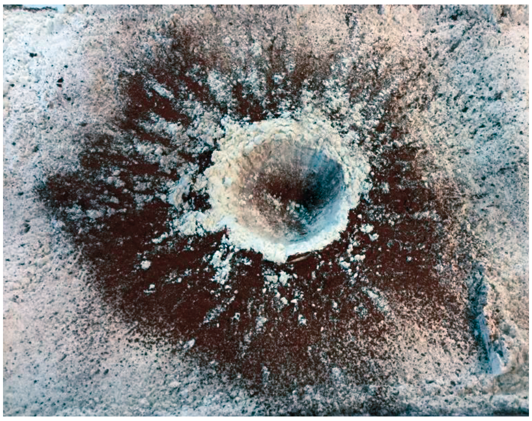
Equipment needed

- deep baking tray or cardboard box (at least 30 x 30 cm)
- flour (enough to fill the above container to a depth of 5 cm)
- cocoa powder (for a thin ejecta 'blanket' layer)
- wooden ball (impactor c.1.75 radius; 30g)

MarsQuake

The landing is prepared by slowly pouring the flour (or sand) into the container; the flour does not want to be too compressed so a sieve should be used if possible. Shift the container from side to side to evenly distribute the flour. The flour should be to a depth of at least 5 cm.





You can investigate the patterns of impact craters in greater detail by tracking the ejecta by using a fine powder that's a different colour to the flour; for example cocoa powder, custard powder or powdered paint. Some of the coloured powder is sieved onto the surface of the flour when preparing the impact area; this will allow the ejecta and rays thrown up from the impact event to be seen and measured.

Procedure

Present your students with a collection of different spherical objects. An introductory investigation will allow them to select which object they consider to be the 'best' impactor.

Select just one of the objects; the same object will used as an impactor throughout the investigation.

The best impactor has been found to be a wooden ball (the density is a key factor) of radius around 1.75 cm and mass 30 g.

The impactor will be dropped from various heights, which simulates different impact speeds (assuming the impactor doesn't reach terminal velocity).

An element of experimental design can be introduced: get the students to plan their own heights. A range from 20 cm up to 200 cm with a 20 cm interval works well.

There is a critical height above which your students will obtain 'good' craters. You might want to leave this for your students to find out for themselves or you could tell them. However, they will need to drop their impactors above this height if they are looking for quantative data. For the wooden ball and flour the critical height was experimentally found to be 0.72 m.

An explanation for this phenomenon could be that at a lower drop height there is a point where the force of impact is not strong enough to overcome the particle-particle interaction of the flour granules, and as a result the crater diameter is not made significantly wider with increasing height.

The impact surface is prepared in the same way as the first introductory investigation. The experiment follows the procedures as before: drop the impactor into the container from each of the selected heights, repeat and carefully measure the distance across the centre of each depression in the flour.

Record the height measurements in a data table like the one below.

Height	Diameter of Craters/cm	Average Crater Diameter /cm

Example data table.

Analysis and conclusions — mathematical modelling

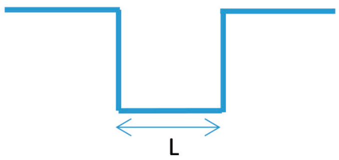
This investigation develops the idea of impactors and crater formation to use energy changes involving potential and kinetic energies to build up two mathematical models. Half of the class could investigate one of the models and then evaluate against the other model.

Two mathematical modelling approaches are presented.

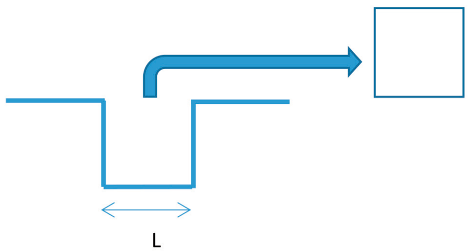
Digging a hole

One approach to modelling of the crater experiments is to consider the formation of a crater as being similar to digging a hole (Byfleet, 2007; Florida State University).

A crater is modelled as a cubic hole, with sides of dimension L, dug into the sand.



Energy considerations would suggest that the energy required in digging this hole will be the same as the gain in potential energy in lifting a similar sized cube of material onto the ground next to the hole.



The volume of the hole, $V = L^3$

Mass of material moved from hole, M = volume x density = $L^3 \times \rho$

Weight of this material, $W = M \times g = L^3 \times \rho \times g$

Gain in potential energy, $E_p = W \times L$

It is suggested that this analysis will be true for any shape of crater; we will just introduce a shaping factor to expand to any shape of crater. This shaping factor will be the same for all collisions within an investigation as long as the shape of crater is consistent throughout the investigation.

A hypothesis could be that at higher velocities there could be more tunnelling or perhaps greater compaction and so different shapes. The consistency of the sand (e.g. wetness, size of grain) could also be a variable that would affect the shaping factor.

$$\text{So, } E_p = W \times L = L^3 \times \rho \times g \times L = L^4 \times \rho \times g \times f_c \text{ where } f_c \text{ is the crater shaping factor}$$

The potential energy for digging the hole will come from the kinetic energy of the ball just before impact and the kinetic energy of the ball will equal the loss in potential energy of the ball during its fall (assuming no losses due to air resistance).

Kinetic energy of ball just before impact, E_k = loss of potential energy of falling ball = m x g x h

In our model we are assuming no energy losses, so all the kinetic energy of the falling ball just before collision will be converted into the potential energy gained by moving the material during the hole digging.

$$m \times g \times h = L^4 \times \rho \times g \times f_c$$

$$\text{So, } h \propto L^4 \text{ or } m \propto L^4$$

Theory therefore predicts that a graph of height of ball drop or mass of ball against (size of crater)^4 should yield a straight line since ρ and the craters' shape are all constant.

Power law

Another approach is to consider that experimental investigations have shown there is a power law relationship between the kinetic energy of the impactor E (=½mv^2) and the resulting crater diameter D (Bunce, 2006; Leicester University).

Crater theory suggests that:

$$D = kE^n \text{ where k and n are (non-integer) constants}$$

Taking natural logs of the above equation gives:

$$\ln(D) = n\ln(E) + \ln(k)$$

So a graph of ln(D) (along the y-axis) against ln(E) (along the x-axis) graph will produce a linear relationship with n as the gradient of the graph, and k as the intercept.