Earthquake magnitude

The magnitude of an earthquake is a number that relates to the amplitude of the earthquake. Earthquake magnitude scales are logarithmic (i.e. a 1 unit increase in magnitude corresponds to a 10 fold increase in amplitude). Scientists can only estimate the true magnitude of an earthquake by measuring its effects, this leads to earthquakes appearing to have different magnitudes depending on what method is used for estimating the magnitude and which datasets have been used to make this estimate. Press reporters love the Richter scale and will report any earthquake magnitude as a 'magnitude on the Richter scale', however for any large earthquake that has made the news it is very unlikely that a Richter (or Local magnitude) is an appropriate scale. Due to the complexities of the calculations the reported magnitude of events can change as more data gets analysed. For the devastating Dec 24th 2004 event in Sumatra the original magnitude of Mw 9.0 was later recalculated to Mw 9.3 as more data was analysed.

Richter magnitude (Local Magnitude M_L)

Originally earthquake magnitudes were based on the amplitude of ground motion displacement as measured by a standard seismograph. The best known of these is the Richter Magnitude which was defined for local earthquakes in southern California

$$M_L = \log A + 2.56 \log D - 1.67$$

Where A is the measured ground motion (in micrometers) and D is the distance from the event (in km). This is still used for measuring the magnitude of shallow events at distances less than 600 km (today called the Local Magnitude). For events larger than magnitude 8 this scale saturates and gives magnitude estimates that are too small.

Body wave magnitude M_b

For earthquakes measured at distances greater than 600 km magnitude can be estimated from the formula.

$$M_b = \log(A/T) + \sigma(D,h)$$

Where A is the maximum amplitude (in micrometers) of the P waves measured at period T (generally about 1second) and σ is a calibration term (in the range 6–8) that depends on distance from the event D and depth of the event h (tables of σ are used).

Surface wave magnitude Ms

For shallow earthquakes (i.e. ones that generate surface waves) magnitudes can be estimated using the formula.

$$Ms = \log(A/T) + 1.66\log\Delta + 3.3$$

Where A is the maximum amplitude (in micrometers) of the Rayleigh waves, T is the period (usually about 20 seconds) and Δ is the distance (in degrees)

Moment magnitude

Earthquakes occur when the ground ruptures.   Stresses build up over time (usually caused by the slow movements of tectonic plates) and eventually a piece of the Earth's brittle crust deep under ground breaks (the technical term is ruptures). This rupture then grows until eventually a large area has shifted (the rupture propagates at a velocity of 2–3km/sec). The magnitude of the earthquake is related to the size of the rupture.

$$\text{Seismic moment (Mo)} = \mu * \text{rupture area} * \text{slip length}$$

where μ is the shear modulus of the crust (approx 3×10^{10} N/m)

$$\text{Moment magnitude (Mw)} = 2/3\log(Mo) - 6.06$$

Nowadays the moment magnitude scale is the one used by seismologists to measure large earthquakes. The historic Richter magnitude is calculated by measuring the deflection on a seismometer corrected for distance from the event. Richter magnitudes underestimate the size of large events and are no longer used. However the constants used in the definition of Moment magnitude (Mw) were chosen so that the magnitude numbers for Richter and Moment magnitudes match for smaller events.   For the largest events (the Mw 9.3 event on Dec 26th 2004) the rupture area can be 1200 km long by 100 km deep with a slip length of up to 15 m (it had a seismic moment of 1.1×10^{23} Nm)

How can an earthquake have a *negative* magnitude?

Very small events (eg. If *2/3log(Mo)*<6.0 or if logA+2.56logD<1.67 **)** will have a magnitude less than zero. In practice earthquakes this small, although quite numerous, are usually only recorded and located in very small scale studies (e.g. studying rockbursts in underground mines)

How is energy related to magnitude?

Seismologists have determined that the energy radiated by an earthquake is a function of both the amplitude of the waves and the duration of the earthquake. A very small earthquake is over in less than a second while for the largest events the fault may continue to slip for more than 5 minutes.

For each unit increase in magnitude M, the amplitude increases by a factor of 10.
Empirical studies have found that:

$$\text{Energy is proportional to } 10^{(1.5M)}$$

Consider the energy (E1) from a magnitude M and from (E2) from magnitude M+1

$$E2/E1 = (10^{(1.5M + 1.5)})/(10^{1.5M}) \qquad E2/E1 = 10^{1.5} = 32$$

Thus, for each unit increase in magnitude, the energy increases by a factor of 32.
For two units of magnitude, the increase is a factor of 10^3 or one thousand.

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Seismic energy:

Both the magnitude and the seismic moment are related to the amount of energy that is radiated by an earthquake. Richter, working with Dr. Beno Gutenberg, early on developed a relationship between magnitude and energy. Their relationship is:

$$\log E = 4.8 + 1.5M$$

giving the energy E in joules from the magnitude M.

Earthquake energy as a function of magnitude		
Magnitude	Energy in joules	Notes
-3.0	2	1 kg dropped 20 cm
-2.0	63	
-1.0	2000	100 kg person jumps down 2 m
0.0	6.3×10^4	
1.0	2.0×10^6	
2.0	6.3×10^7	Only felt nearby
3.0	2.0×10^9	Energy from 50 litres of petrol
4.0	6.3×10^{10}	Often felt up to 10's of miles away
5.0	2.0×10^{12}	Energy from 50 000 litres of petrol
6.0	6.3×10^{13}	3.3 Hiroshima-size A bombs
7.0	2.0×10^{15}	
8.0	6.3×10^{16}	1–2 earthquakes this size each year
9.0	2.0×10^{18}	Total annual energy use of UK

Note that E is not the total 'intrinsic' energy of the earthquake, transferred from sources such as gravitational energy or to sinks such as heat energy. It is only the amount radiated from the earthquake as seismic waves, which ought to be a small fraction of the total energy transferred during the earthquake process.

More recently, Dr. Hiroo Kanamori came up with a relationship between seismic moment and seismic wave energy.

Energy = (Moment)/20 000

For this moment is in units of Nm, and energy is in units of joules.

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